Noise-based electronic refrigerators—how practical?

Electronic-based refrigerators constitute one class of electrical elements that often work based on the principles of thermoelectric effects. These devices, which are much quieter than their compressor-based counterparts, are gradually gaining more popularity and are sometimes considered replacement candidates for today's compressor-based refrigerators. Over the past few decades, several books and numerous articles have been published on the properties of electronic-based refrigerators and these efforts are still ongoing. Today, several research groups around the globe are investigating the properties of thermoelectric effects and are trying to develop a high-efficiency refrigerator based on these effects.

REZA NAVID

We will discuss the possibility of building an electronic refrigerator based on the properties of noise by introducing a noise-based refrigerator suitable for integrated circuits. We will then focus on the implementation and the efficiency of such a device. Although the efficiency of such a device is too low to be practically useful, this discussion is educational and helps to gain a better understanding of noise in electronic devices.

Now hear this

To understand how a noise-based refrigerator works, let us start by describing a thought experiment. Let us assume, as shown in Fig. 1(a), that the two systems A and B are connected through an electrical port. Let us further assume that the input resistance of both systems is R. In

© ARTVILLE

Digital Object Identifier 10.1109/MPOT.2008.928013

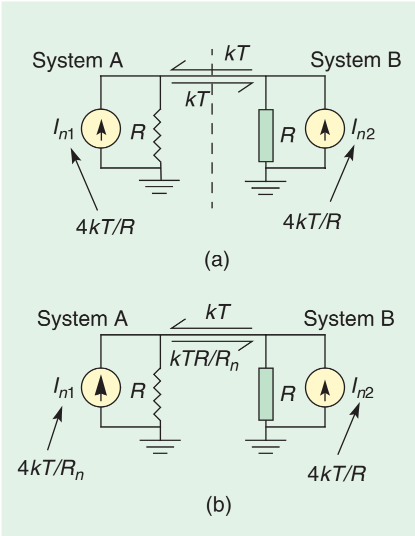

Fig. 1 The thought experiment that explains how a noise-based refrigerator works. (a) Both systems in equilibrium and no cooling effect. (b) System A driven into non-equilibrium and system B is getting cooled down.

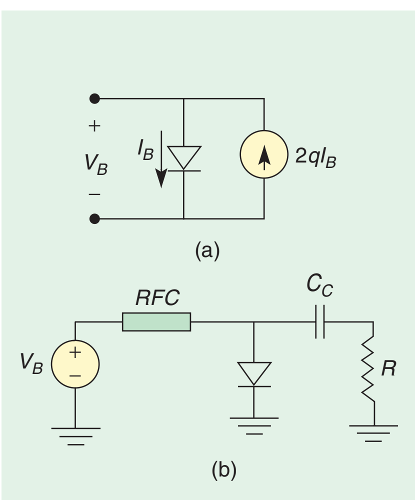

Fig. 2 (a) Noise in a forward-biased PN junction diode and (b) the experiment for assessing the efficiency of the noise-based refrigerator.

general, the noise associated with each of these systems can be modeled by a noise current source connected in parallel to the electrical port. Each of these noise current sources injects a finite amount of energy into the resistor on the other side of the dashed line. Since, under thermal equilibrium conditions, the net flow of energy must be zero, one can conclude that the stochastic properties of the two current noise sources have to be identical in thermal equilibrium (one of the implications of the famous fluctuation dissipation theorem of thermodynamics). Quantitatively, the power spectral density of the current noise source associated with both systems is white and has an amplitude of $4kT/R$. Each of the systems will inject an energy of kT W/Hz into the other system and the net flow of energy is zero, as expected.

Imagine that we drive one of the systems shown in Fig. 1(a) into non-equilibrium conditions. In general, this can alter both the noise power spectral density and the input resistance of this system. Fig. 1(b) shows the case when system A is driven into non-equilibrium while system B is still in its thermal equilibrium state. In this second thought experiment, suppose that the input resistances of the two systems are still the same as before (R) but the noise power associated with system A is smaller than the previous case. For example, let us assume that the noise power spectral density of system A is white and has an amplitude of $4kT/R_n$,where $R_n > R$.

Under these assumptions, the noise current source I_{n1} has a total load resistance of $R/2$ and delivers a total power of

$$P_{In1} = I^2 R_{\text{load}} = \frac{4kT}{R_n} \cdot \frac{R}{2} = 2kTR/R_n.$$

Since the two resistors in systems A and B are identical, this power will be split equally between the two. Therefore, the total power delivered from I_{n1} to each resistor R is $P_{In1toB} = kTR/R_n$. The total power delivered from I_{n2} to each resistor is $P_{In2toA} = kTR/R = kT$. Note that the total power dissipated in the two resistors is identical and equal to $kT(1 + R/R_n)$. However, the power delivered to resistor R in system B by the noise source of system A (kTR/R_n) is different from the power delivered to resistor R in system A by the noise source of system B (kT). This means system A injects kTR/R_n W/Hz into system B, while system B injects kT W/Hz into system B. Since we already assumed that $R_n > R$, we conclude that there will be a net flow of energy from system B into system A whose value is $kT(R_n - R)/R_n$ W/Hz. Thus, system B is being cooled down as thermal energy is being sucked out of this system. This is the essence of the proposed noise-based refrigerator.

You're getting colder

As mentioned in the previous section, for implementing a noise-based refrigerator we first need a system whose non-equilibrium noise power is smaller than its equilibrium noise power predicted by the fluctuation dissipation theorem. Fortunately, such systems are readily available. MOS transistors and PN junction diodes are two examples of such systems. We use the latter one to describe a possible implementation of a noise-based refrigerator because it is easier to describe and has a better efficiency.

Fig. 2(a) shows a forward-biased PN junction diode. Detailed analysis of noise in PN junctions shows that if the diode is forward biased with $V_B >> kT/q$, its noise can be modeled by a noise current source whose power spectral density follows the shot noise formula given by

$$\frac{I_n}{\Delta f} = 2qI_B \tag{1}$$

where I_B is the DC current in the diode.

Let us now calculate the effective small-signal resistance of the diode at the bias point. The I-V characteristic of an ideal diode is described by:

$$I_B = I_s\left(\exp\left(\frac{V_B q}{kT}\right) - 1\right) \tag{2}$$

where I_s is the diode's saturation current. The equivalent small signal resistor is defined as

$$R_{eq} = \frac{dV_B}{d\,I_B} \tag{3}$$

where we took advantage of the fact that noise amplitude is small and the system can be considered linear. Using (2) and (3) and noting that $V_B >> kT/q$, one can easily show

$$R_{\text{eq}} = \frac{kT}{qI_B}. \tag{4}$$

Omitting I_B between (1) and (4) will result in an equation which relates the system noise to its equivalent small signal resistance:

$$\frac{I_n}{\Delta f} = 2kT/R_{\text{eq}}. \tag{5}$$

This means that the non-equilibrium noise in forward-biased PN junctions is half the value predicted by the fluctuation dissipation theorem. Therefore, a forward biased PN junction diode can serve as system A in Fig. 1(b) with $R_n = 2R > R$ as required.

Using these results, a possible implementation of the proposed noise-based refrigerator is shown in Fig. 2(b). Here, *RFC* (radio frequency choke) is a large inductor that is used for biassing and is ideally noiseless. The coupling capacitor C_C blocks any DC current that would otherwise flow into R and inadvertently warm it up. The value of C_C also determines the lower corner of the bandwidth over which the diode and the resistor R exchange noise power.

As was discussed earlier, in the experiment of Fig. 2(b), R_n has a value twice that of R. Therefore, in steady state, the temperature of resistor R has to be half of the temperature of the diode if we assume that the resistor R is thermally isolated. This is because, in steady state, the net flow of noise energy across the dashed line should be zero. Assuming that the diode is at room temperature (300 K), the resistor will be at 150 K or −123 °C! Note that this dramatic cool-down of resistor R, although intuitively implausible, does not defy the second law of thermodynamics because V_B is constantly injecting energy into the system.

A person familiar with circuit design will promptly recognize that configurations like the one shown in Fig. 2(b) are routinely built in various circuits. One might then wonder how come he/she has never seen crystalline ice on his circuit. To find out the answer, we need to evaluate the efficiency of our refrigerator.

Sufficient efficiency?

The efficiency of the noise-based refrigerator introduced in this article can be assessed using some simple principles and typical numbers. We would like to find the best possible performance achievable using this refrigerator.

Let us first assume that the experiment shown in Fig. 2(b) is performed using discrete elements. The first thing to calculate is the flow of noise energy, which needs to be maximized to get the best performance. The maximum power delivery criterion dictates that the flow of energy is maximized for $R = R_{eq}$. Under this condition, the net flow of energy from the resistor to the diode is $kT/2$ per each Hz of bandwidth. In order to maximize the energy flow, one needs to maximize the system bandwidth. Given that the system is built using discrete elements, we can assume a maximum bandwidth of 1 GHz to calculate the upper limit on the efficiency of the system. The numerical value of the net energy flow at the room temperature (300 K) is then given by:

$$E_{\mathrm{FNet}} = \frac{kT}{2} \times BW = 2.1 \cdot 10^{-21} \times 10^{9} = 2.1 \cdot 10^{-12}\ \mathrm{J/s}. \quad (6)$$

As a measure of efficiency, let us calculate how long it would take to cool down the resistor by 1 °C if the resistor is thermally isolated. Assuming that the resistor is made out of carbon, it will have a heat capacity and mass density of about 711 J/Kg°C and 2,000 Kg/m^3, respectively. The typical physical size for a discrete resistor is 0.5 cm × 1 mm × 1 mm, giving rise to a total volume of $5 \cdot 10^{-9}$m^3. Using these numbers, one can calculate the amount of heat that should be extracted from the resistor in order to decrease its temperature by 1°C:

$$E = 711 \times 2000 \times 5 \cdot 10^{-9} = 7.11 \cdot 10^{-3}\ \mathrm{J}. \quad (7)$$

Assuming perfect thermal isolation of the resistor, the required time for changing its temperature by 1°C is found by dividing (7) by (6):

$$t = \frac{7.11 \cdot 10^{-3}}{2.1 \cdot 10^{-12}} = 3.4 \cdot 10^{9}\ \mathrm{s}, \quad (8)$$

which is approximately 107 years! This is an unreasonably long time to wait in order to see the effect of the noise cooling phenomena described.

There is also a heat flow between the resistor and the environment, making it even more difficult to observe the effect of our noise-based refrigerator. This answers the question that we posed earlier—this phenomenon is so inefficient that its effect is unlikely to be detected or observed.

Read more about it

- I.B. Cadoff and E. Miller, *Thermoelectric Materials and Devices*. New York: Reinhold, 1960.

About the author

Reza Navid (rnavid@gloworm.stanford.edu) is the senior member of technical staff at technology licensing company Rambus Inc.

0278-6648/08/$25.00 © 2008 IEEE